2016

(2nd Semester)

BACHELOR OF COMPUTER APPLICATIONS

Paper No. : BCA-202

[Mathematics—II (Discrete Mathematics)]

(New Course)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. (a) For any a, b, c, d in a lattice $(A, \leq)$, if $a \leq b$ and $c \leq d$, then show that $a \vee c \leq b \vee d$ and $a \wedge c \leq b \wedge d$. 5

   (b) For any a and b in a Boolean algebra, show that

   $$\overline{a \vee b} = \overline{a} \wedge \overline{b} \text{ and } \overline{a \wedge b} = \overline{a} \vee \overline{b}$$ 5

2. *(a)* Define the following terms : 5

   (i) Disjoint set

   (ii) Boolean sum

   (iii) Universal set

   (iv) Complemented lattice

   (v) Boolean lattice

   (b) Let $(A, \vee, \wedge, -)$ be a finite Boolean algebra. Let b be any non-zero element in A and $a_1, a_2, \cdots, a_k$ be all the atoms of A such that $a_i \le b$. Show that

$$b = a_1 \vee a_2 \vee \cdots \vee a_k$$ 5

UNIT—II

3. *(a)* Give the truth table for the following : 2+3

$$(Q \wedge (P \to Q)) \to P$$

$$(P \to Q) \rightleftarrows (\rceil P \vee Q)$$

   (b) Define tautology. Explain with an example. Prove that

$$(P \to Q) \Leftrightarrow (\rceil P \vee Q)$$ 3+2

4. *(a)* Obtain the principal conjunctive normal form of the formula S given by

$$(\urcorner P \to R) \wedge (Q \rightleftarrows P)$$ 5

(b) Show that

$$(\urcorner P \wedge (\urcorner Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$$ 5

UNIT—III

5. *(a)* If $(1-x)^{-n} = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$, find the value of $a_0 + a_1 + a_2 + \cdots + a_n$. 5

(b) Find the coefficient of x^4 and x^6 in the expansion of $\left(\frac{1-x}{1+x}\right)^2$. 5

6. *(a)* Find the number of ways to paint 12 offices so that 3 of them are green, 2 of them are yellow and the remaining ones white. Also define combinations. 65440 4+1

(b) Find the number of different words beginning with C which can be performed by using all the letters of the word 'COMPUTER'. 5040 5

UNIT—IV

7. (a) If G is a finite group and H is a subgroup of G, then prove that $O(H)$ divides $O(G)$. 5

(b) In a semi-group S, $x^2y = y = yx^2$, $\forall\ x, y$, then show that S is abelian. 5

8. (a) Let $(A, *)$ be a group and B is a finite subset of A, show that $(B, *)$ is a subgroup of $(A, *)$ if $*$ is a closed operation on B. 5

(b) Show that a subgroup of index 2 in a group G is a normal subgroup of G. 5

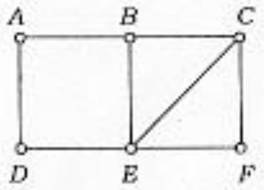
UNIT—V

9. (a) If there is a path from vertex v_1 to vertex v_2 in a graph with n vertices, show that there is a path of no more than $n-1$ edges from vertex v_1 to vertex v_2. 5

(b) Show that there is always a Hamiltonian path in a directed complete graph. 5

10. (a) In any connected planar graph, show that $v - e + r = 2$, where v, e and r are the number of vertices, edges and regions of the graph respectively. 5

(b) Define connected graph and tree. Write down the spanning trees of the following graph : 5



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