2 0 1 5

(2nd Semester)

BACHELOR OF COMPUTER APPLICATIONS

Paper No. : BCA–202

[Mathematics—II (Discrete Mathematics)]

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—I

(*Marks* : 15)

I. Put a Tick (✓) mark against the correct answer in the brackets provided :   1×10=10

   1. If $A = \{5, 7, 9, 11, 13, 15\}$ and $B = \{x : x = 2n + 1,$ $2 \le n \le 7, n \in \mathbf{N}\}$, then $B - A$ is equal to

     (a) $\{9, 11, 13, 15\}$ (   )

     (b) ϕ (   )

     (c) $\{5, 7, 9\}$ (   )

     (d) B (   )

2. A self-complemented distributive lattice is called

   (a) Boolean algebra   (   )

   (b) modular lattice   (   )

   (c) complete lattice   (   )

   (d) self-dual lattice   (   )

3. Let p denote "He is rich" and let q denote "He is happy". Then the statement formula $q \rightarrow \neg p$ is equivalent to

   (a) if he is rich, then he is unhappy   (   )

   (b) it is necessary to be poor in order to be happy   (   )

   (c) he is neither rich nor happy   (   )

   (d) to be poor is to be unhappy   (   )

4. Which of the following propositions is a tautology?

(a) $(p \vee q) \to p$ (   )

(b) $p \vee (q \to p)$ (   )

(c) $p \vee (p \to q)$ (   )

(d) $p \to (q \to p)$ (   )

5. The number of different permutations of the word 'BANANA' is

(a) 720   (   )

(b) 120   (   )

(c) 60   (   )

(d) 360   (   )

6. The 3rd term in the expansion of $\left(3x - \dfrac{y^3}{6}\right)^4$ is

(a) $\dfrac{2}{3} xy^3$ (   )

(b) $\dfrac{12}{5} xy^{-2}$ (   )

(c) $-\dfrac{3}{2} x^3 y^5$ (   )

(d) $\dfrac{3}{2} x^2 y^6$ (   )

7. The set of integers $\mathbb{Z}$ with the binary operation $*$ defined as $a * b = a + b + 1$ for $a, b \in \mathbb{Z}$, is a group. The identity element of this group is

(a) -1 (   )

(b) 1 (   )

(c) 0 (   )

(d) 12 (   )

8. A necessary and sufficient condition for a non-empty subset H of a finite group G to be a subgroup is that

   (a) $a \in H$, $b \notin H \Rightarrow a, b \in H$ (   )

   (b) $a \in H$, $b \in H \Rightarrow (a + b) \in H$ (   )

   (c) $a \in H$, $b \in H \Rightarrow ab \in H$ (   )

   (d) $a \in H$, $b \in H \Rightarrow (a - b) \in H$ (   )

9. The total number of edges in a complete graph of n vertices is

   (a) n (   )

   (b) $\dfrac{n(n-1)}{2}$ (   )

   (c) $\dfrac{n}{2}$ (   )

   (d) $n^2 - 1$ (   )

10. If a graph G is bipartite, then the chromatic number (χ) of G is

    (a) 1   (   )

    (b) 3   (   )

    (c) 0   (   )

    (d) 2   (   )

II. Tick (✓) either True or False :   1×5=5

11. Let $(P(A), \subseteq)$ be a poset where A is any non-empty finite set, $P(A)$ power set of A and $\subseteq$ is 'set inclusion'. Then the least element of $(P(A), \subseteq)$ is any singleton set.

                        True ( ) / False ( )

12. The contrapositive of the statement 'If $f(2) = 0$, then $f(x)$ is divisible by $(x-2)$' is '$f(x)$ is divisible by $(x-2) \Rightarrow f(2) \neq 0$'.

                        True ( ) / False ( )

13. If $^nC_p = {}^nC_q$, then $p = q$ or $p + q = n$.

$\qquad$ True ( ) / False ( )

14. Burnside's theorem states that if G is a finite group of order $p^a q^b$ where p and q are any integers, and a and b are non-negative integers, then G is solvable.

$\qquad$ True ( ) / False ( )

15. Sum of the degrees of all regions in a map is equal to twice the number of edges in the corresponding graph.

$\qquad$ True ( ) / False ( )

=5

SECTION—II

(*Marks* : 10)

III. Answer the following questions :   2×5=10

1. Let $N = \{1, 2, 3, ...\}$ be ordered by divisibility. State with brief justification whether each of the following subsets of N is linearly (totally) ordered :

(a) {24, 2, 6}

(b) {3, 15, 5}

(c) $N = \{1, 2, 3, ...\}$

(d) {2, 8, 32, 4}

2. Using truth table, prove that the following argument is valid :

$$p \rightarrow \neg q, r \rightarrow q, r \vdash \neg p$$

10

3. Find the number of different words beginning with P which can be formed by using all the letters of the word 'PERMUTATION'.

4. Define cosets with an example.

5. Draw a graph having—

   (a) one cutpoint and one bridge;

   (b) no cutpoint and no bridge.

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2 0 1 5

(2nd Semester)

BACHELOR OF COMPUTER APPLICATIONS

Paper No. : BCA–202

[Mathematics—II (Discrete Mathematics)]

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks
for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* In a survey of 60 people, it was found that 25 people read newspaper *A*, 26 read newspaper *B*, 26 read newspaper *C*, 9 read both *A* and *C*, 11 read both *A* and *B*, 8 read both *B* and *C*, and 3 read all the three newspapers. Find—

    (i) the number of people who read at least one of the newspapers;

    (ii) the number of people who read exactly one newspaper.   5

(b) Define a finite Boolean algebra and show that D_{70} is a finite Boolean algebra with the partial order of divisibility, where D_n is defined as the set of all positive divisors of n. 5

2. (a) In a survey of 100 students, the number of students studying the various languages is found as English only 18; English but not Hindi 23; English and Sanskrit 8; Sanskrit and Hindi 8; English 26; Sanskrit 48 and no language 24. Find—

(i) how many students are studying Hindi;

(ii) how many students are studying English and Hindi both. 5

(b) Let $S = \{a, b, c\}$ and $L = P(S)$. Prove that $(L, \subseteq)$ is isomorphic to D_{42}. Here $P(S)$ is defined as the power set of S and D_n the set of all positive divisors of n. 5

UNIT—II

3. (a) Without constructing the truth table, show the following equivalence

$$(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) = r$$ 5

(b) Obtain the principal disjunctive normal form of the formula

$$(p \to (q \wedge r)) \wedge (\neg p \to (\neg q \wedge \neg r))$$ 5

4. (a) Without constructing the truth table, show the following equivalence : 5

$$(p \vee q) \wedge (\neg p \wedge (\neg p \wedge q)) \equiv (\neg p \wedge q)$$

(b) Obtain the principal conjunctive normal form of the formula

$$p \vee (\neg p \rightarrow (q \vee (\neg q \rightarrow r)))$$ 5

UNIT—III

5. (a) A committee of 5 is to be selected from among 6 boys and 5 girls. Determine the number of ways of selecting the committee if it is to consist of at least one boy and one girl. 5

(b) If the 4th term in the expansion of

$$\left(ax + \frac{1}{x}\right)^n$$

is $\frac{5}{2}$, find the values of n and a. 5

6. (a) In an examination paper, there are two groups each containing 7 questions. A candidate is required to attempt 9 questions but not more than 5 questions from each group. In how many ways can 9 questions be selected? 5

(b) If x^p occurs in the expansion of

$$\left(x^2 + \frac{1}{x}\right)^{2n}$$

prove that its coefficient is

$$\frac{(2n)!}{\left(\frac{4n-p}{3}\right)!\left(\frac{2n+p}{3}\right)!}$$

5

UNIT—IV

7. (a) Show that the set $\mathbb{Z}$ of all integers is an Abelian group with $*$ defined by

$$a * b = a + b + 2$$

5

(b) Define normal subgroup. Suppose that N and M are two normal subgroups of G and that $N \cap M = \{e\}$. Show that every element of N commutes with every of M. 5

8. (a) Show that the set of all matrices

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

a and b being non-zero reals, is a group under matrix multiplication. 5

(b) Define group homomorphism. Let f be a homomorphic mapping of a group G into a group J. Let $f(G)$ be the homomorphic image of G in J. Then show that $f(G)$ is a subgroup of J. 5
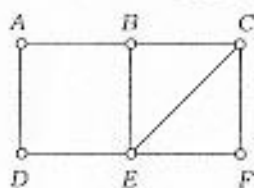
UNIT—V

9. (a) Define the following terms with examples : 5

   (i) Complete graph

   (ii) Planar and non-planar graphs

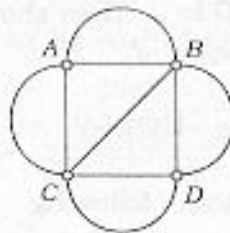(b) Write the adjacency matrix and the incidence matrix for the following graph : 5
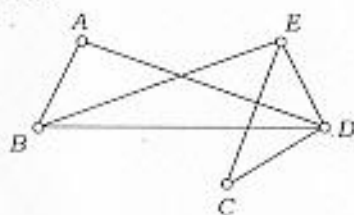
10. (a) Define tree. Write down all the spanning trees of the following graph : 5

(Turn Over)

(b) Define Hamiltonian graph. Check whether the following graph has Hamiltonian circuit or not :

Give sufficient reason to support your answer. 5

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