Professional Course Examination, 2021

(2nd Semester)

BACHELOR OF COMPUTER APPLICATIONS

Course : BCA/2/CC/08 (New)

(Discrete Mathematics)

Full Marks : 75

Time : 3 hours

INSTRUCTIONS TO CANDIDATES

(Please read the instructions carefully before you start writing your answers)

1. Questions should be attempted as per instructions.

2. Do not copy the Questions. Indicate the Part and Section with Question No. clearly while attempting the answer.

3. For Multiple choice answer, candidate should indicate the **Question No., Sub. No., (if any) and the correct answer. For example :**

    1.   Name the State capital of Mizoram.

       (a) Lunglei

       (b) Aizawl

       (c) Champhai

    Candidate should provide answer as—Q. No. *1* : *(b) Aizawl*
    [Candidate should **avoid** writing only *(b)*]

4. The figures in the margin indicate full marks for the questions.

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(*Marks* : 25)

SECTION—I

(*Marks* : 15)

Choose the correct answer from the options provided :   1×10=10

1. Two sets A and B are said to be disjoint if

   (a) $A \quad B$

   (b) $A \quad B$

   (c) $A \quad B \quad B$

   (d) $A \quad B \quad A$

2. A set A is a proper subset of a set B if

   (a) $A \quad B$

   (b) $A \quad B$

   (c) $A \quad B$

   (d) Both *(a)* and *(b)*

3. Which of the following proportions is tautology?

   (a) $(p \quad q) \quad q$

   (b) $p \quad (p \quad q)$

   (c) $p \quad (p \quad q)$

   (d) Both *(b)* and *(c)*

4. A statement formula which is true regardless of the truth values of the statement is

   (a) tautology

   (b) connective

   (c) equivalence

   (d) normal form

5. The middle term in the expansion of $(3 \quad x)^6$ is

(a) $640x^2$

(b) $540x^3$

(c) $480x^3$

(d) $260x^2$

6. The number of permutations of n different objects taken r at a time, where repetition is allowed, is

(a) r^n

(b) rn

(c) n/r

(d) n^r

7. The inverse of 3 modulo 7 is

(a) −1

(b) −2

(c) −3

(d) −4

8. Which positive integer less than 21 is relatively prime to 21?

(a) 18

(b) 19

(c) 21

(d) 24

9. A graph in which every edge is directed is called

(a) digraph

(b) mixed graph

(c) simple graph

(d) path connected graph

10. A tree with n vertices has _____ edges.

(a) n

(b) n 1

(c) 0

(d) None of the above

Choose and write the correct answer (True/False) :   1×5=5

11. The set of all circles passing through a given point is finite set.

(a) True

(b) False

12. $(P \quad Q)$ is equivalent to $P \quad Q$.

(a) True

(b) False

13. $\sum\limits_{r \ 1}^{6} \, {}^5c_r$ is equal to 25.

(a) True

(b) False

14. The linear combination of gcd $(252, 198)$ 18 is 252 4 198 5.

(a) True

(b) False

15. An open walk in which no vertex appear more than once is called path.

(a) True

(b) False

SECTION—II

(*Marks* : 10)

Answer the following questions :   2×5=10

16. *(a)* Given $A = \{x : x$ is an integer and $1 \le x \le 5\}$, $B = \{3, 4, 5, 15\}$. Find $A \cup B$ and $A \cap B$.

OR

(b) Show that D_{20} is not a finite Boolean algebra with the partial order of divisibility.

17. *(a)* Write the truth table of $\sim q \to (\sim p \vee q)$.

OR

(b) Write a truth table of $(p \wedge q) \vee r$.

18. *(a)* Expand $(x^2 - 2y)^5$ by binomial theorem.

OR

(b) If ${}^n P_r = 720$ and ${}^n C_r = 120$, then find r.

19. *(a)* Solve $15x \equiv 7 (\mathrm{mod}\, 32)$.

OR

(b) Suppose a and c are relatively prime integers and b is an integer such that b/c. Prove that gcd $(a, b) = 1$.

20. *(a)* What is multigraph?

OR

(b) Define a regular graph and a tree.

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

Answer the following questions : 10×5=50

1. *(a)* Define a Boolean algebra and write its basic properties. 5

(b) For any a and b in a Boolean algebra, show that

$$\overline{a \cdot b} = \bar{a} + \bar{b} \text{ and } \overline{a + b} = \bar{a} \cdot \bar{b}$$ 5

OR

2. *(a)* If A, B and C are sets, prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. 5

(b) In a survey of 100 students, the number of students studying the various languages is found as—English only 18, English but not Hindi 23, English and Sanskrit 8, Sanskrit and Hindi 8, English 26, Sanskrit 48 and no language 24. Find—

(i) how many students are studying Hindi;

(ii) how many students are studying English and Hindi both. 5

3. *(a)* Show that $(P \wedge Q) \vee (\neg P \wedge (\neg P \wedge Q)) \Leftrightarrow (\neg P \wedge Q)$. 5

(b) Obtain disjunctive normal form of $(P \wedge Q) \rightleftharpoons (P \vee Q)$. 5

OR

4. *(a)* Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$. 5

(b) Show that $(P \rightarrow Q) \wedge (Q \rightarrow R) \Rightarrow P \rightarrow R$. 5

5. *(a)* In how many ways can a vowel, a consonant and a digit be chosen out of the 26 letters of the alphabet and the 10 digits? 3

(b) Explain the two fundamental principles of counting. 2

(c) A committee of 5 is to be formed out of 6 men and 4 ladies. In how many ways can this be done, when—

(i) at least 2 ladies are included;

(ii) at most 2 ladies are included? 5

6. *(a)* Find the coefficient of x^5 in the expansion of $(x - 3)^8$.   5

(b) Find the term independent of x in the expansion of $\left(x - \dfrac{2}{x}\right)^{10}$.   5

7. *(a)* State and prove Euclid's lemma.   5

(b) Prove that—

   (i) if p is prime, then $\phi(p) = p - 1$;

   (ii) if p is prime and $n \geq 1$, then $\phi(p^n) = p^n - p^{n-1}$;

   (iii) $\phi(n)$ counts the elements in $\{1, 2, ..., n - 1\}$ which are multiple mod n.   5

OR

8. *(a)* State and prove Euler's theorem.   5

(b) If S is a non-empty subset of N then there is an $m \in S$ such that $m \leq x$ for all $x \in S$. Prove that S has the smallest element.   5
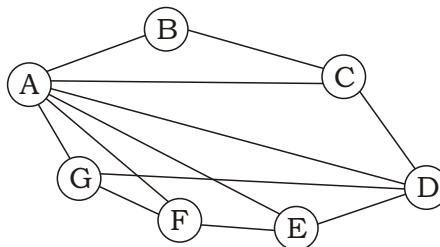
9. *(a)* Explain a graph and a subgraph using relevant examples.   5

(b) What are the necessary conditions for bipartite graph? Explain with relevant graph figure.   5

OR

10. *(a)* Explain minimum spanning tree using examples.   5

(b) What is the formula for adjacency matrix in a graph? Obtain the adjacency matrix from the following graph :   5

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