

Professional Course Examination, May 2023

(2nd Semester)

BACHELOR OF COMPUTER APPLICATIONS

(Discrete Mathematics)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(PART : A—OBJECTIVE)

(Marks : 25)

SECTION—I

(Marks : 15)

A. Tick (✓) the correct answer in the brackets provided : $1 \times 10 = 10$ 1. Two sets A and B are said to be intersecting if

(a) $A \cap B = \phi$ ()

(b) $A \cap B \neq \phi$ ()

(c) $A \cup B = \phi$ ()

(d) $A \cup B \neq \phi$ ()

2. If A and B are two sets such that $n(A) = 24$, $n(B) = 22$ and $n(A \cap B) = 8$, then $n(B - A)$ is

(a) 10 ()

(b) 12 ()

(c) 14 ()

(d) 16 ()

3. In the conditional statement, $P \rightarrow Q$, the statement P is called

(a) hypothesis ()

(b) conclusion ()

(c) antecedent ()

(d) consequent ()

4. Which of the following is **not** a statement?

(a) Close the door ()

(b) The earth is round ()

(c) India is a country. ()

(d) $4 + 7 > 15$ ()

5. $0! = ?$

(a) 0 ()

(b) -1 ()

(c) 1 ()

(d) None of the above ()

6. How many permutations of the letters of the word 'APPLE' are there?
- (a) 30 ()
- (b) 40 ()
- (c) 50 ()
- (d) 60 ()
7. The value of x satisfying $3x \equiv 2 \pmod{5}$ is
- (a) 2 ()
- (b) 1 ()
- (c) 4 ()
- (d) 3 ()
8. The linear combination of $\gcd(252, 198) = 18$ is
- (a) $252*4 - 198*5$ ()
- (b) $252*5 - 198*4$ ()
- (c) $252*5 - 198*2$ ()
- (d) $252*4 - 198*4$ ()
9. A graph is a set of points, called
- (a) lines ()
- (b) fields ()
- (c) edge ()
- (d) nodes ()

10. Self loops are counted

(a) once ()

(b) twice ()

(c) thrice ()

(d) multiple times ()

B. Indicate whether the following statements are *True (T)* or *False (F)* by putting a Tick (✓) mark in the brackets provided : 1×5=5

1. In a Boolean algebra B , for all x, y in B , $x \cdot (x + y)$ is equal to x .

(T / F)

2. The proposition $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$ is a tautology.

(T / F)

3. If ${}^nP_r = 720$, ${}^nC_r = 120$, then r is equal to 3.

(T / F)

4. The inverse of 3 modulo 7 is -2.

(T / F)

5. A graph G is bipartite, then the chromatic number of G is 3.

(T / F)

SECTION—II

(Marks : 10)

C. Answer the following questions :

2×5=10

1. (a) A survey shows that 73% of the Indians like apples, whereas 65% like oranges. What percentage of Indians like both apples and oranges?

OR

- (b) In a Boolean algebra B , show that $a + 1 = 1$.

2. (a) Write the truth table of $\sim(\sim p \wedge q)$.

OR

- (b) Write a truth table of $p \rightarrow (q \wedge \sim p)$.

3. (a) Expand $(3x + 2y)^4$ by binomial theorem.

OR

- (b) If ${}^nC_7 = {}^nC_5$, find n .

4. (a) Suppose a and c are relatively prime integers and b is an integer such that $b|c$. Prove that $\gcd(a, b) = 1$.

OR

- (b) Solve : $6x \equiv 4 \pmod{10}$.

5. (a) What are multigraph and weighted graph?

OR

- (b) Define a planar and non-planar graph.

(PART : B—DESCRIPTIVE)

(Marks : 50)

D. Answer the following questions :

10×5=50

1. (a) A class has 175 students. The following description gives the number of students studying one or more of the subjects in this class :

Mathematics 100, Physics 70, Chemistry 46, Mathematics and Physics 30, Mathematics and Chemistry 28, Physics and Chemistry 23, Mathematics, Physics and Chemistry 18.

Find (i) how many students are enrolled in Mathematics alone, Physics alone and Chemistry alone and (ii) the number of students who have not offered any of these subjects.

3+3=6

- (b) If $A = \{x : x \in N, x \leq 7\}$, $B = \{x : x \text{ is a prime, } x < 8\}$, and $C = \{x : x \in N, x \text{ is odd and } x < 10\}$, verify that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

4

OR

2. (a) In a Boolean algebra $(B, +, \cdot, ')$, prove that

$$(a + b)' = a' \cdot b' \text{ and } (a \cdot b)' = a' + b'$$

5

- (b) Construct the switching table for the switching function f represented by the Boolean expression $xyz + x'(y + z)$.

5

3. (a) Construct a truth table for the statement formula

$$(p \wedge \sim q) \vee (q \wedge (\sim p \vee r))$$

5

- (b) Verify by truth table, $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$.

5

OR

4. (a) Prove that $p \rightarrow q, \sim p \rightarrow q \models q$.

5

- (b) Construct a truth table for the statement :

5

$$(p \rightarrow (q \rightarrow r)) \rightarrow ((p \wedge q) \rightarrow r)$$

5. (a) Find the two middle terms in the expansion of

$$\left(3x - \frac{2}{x^2}\right)^{15}$$

5

- (b) Find the 10th term in the expansion of

$$\left(\frac{a}{b} - \frac{2b}{a^2}\right)^{12}$$

5

OR

6. (a) Find the 5th term from the end in the expansion of

$$\left(x - \frac{1}{x}\right)^{12}$$

5

- (b) If ${}^{20}C_r = {}^{20}C_{r+6}$, find r .

5

7. (a) Find $(26, 118)$ and express it in the form of $26x + 118y$, where $x, y \in \mathbb{Z}$.

5

- (b) Prove that the relation 'congruence modulo m ' is an equivalence relation in the set of integers.

5

OR

8. (a) State and prove Fermat's theorem.

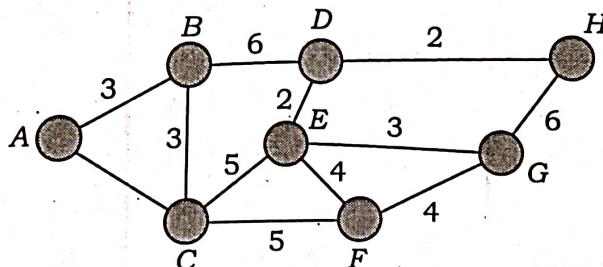
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- (b) Write the statement of Euler's Lemma. Compute $\gcd(803, 154)$.

4

9. (a) Using Kruskal's algorithm, find the minimum spanning tree for the weighted graph of the figure :

6



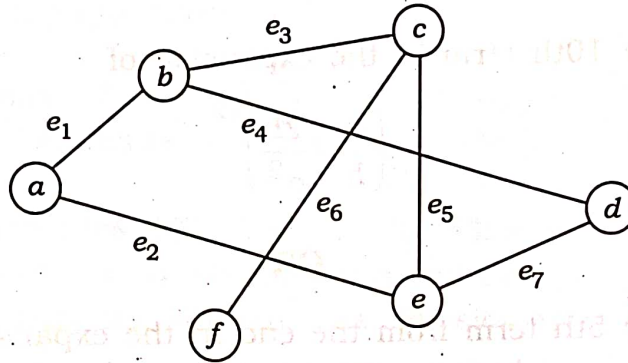
- (b) Prove that a tree with n vertices has $(n-1)$ edges.

4

OR

10. (a) Write the adjacency matrix and incidence matrix for the following graph :

6



- (b) Define tree. Prove that a tree T is always separable.

4

OR

- (a) State and prove Fermat's theorem.
 (b) Write the element of Euler's theorem. Compute $\phi(803, 154)$.
 (c) Using Kruskal's algorithm, find the minimum spanning tree for the weighted graph of the figure.

